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a. Memorandum, AMSRL-CI-LP, undated, Subject: Review for Change in Classification and Distribution, ARL Report No., copy enclosed.

b. Ballistic Research Laboratories Report No. 538, "The Effective Velocity of Escape of the Powder Gas from a Gun", by J. Vinti, April 1945, copy enclosed.

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for Douglas J Kingsley

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Ordnance Research and Development Center Project No. 5265

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2 April 1945

THE EFFECTIVE VELOCITY OF ESCAPE OF THE POWDER GAS FROM A GUN

Abstract

The effective velocity of escape of the powder gas from a gun, as defined by Eq. (1), is a quantity that is independent of the mass of the recoiling parts. A knowledge of this escape velocity permits the calculation of the maximum velocity of free recoil, a quantity of interest in the theory of damped recoil, and therefore of value in the design of a recoil mechanism. Although this escape velocity is independent of the recoiling mass, it depends on other factors, so that the common assumption viz., that it has a universal value, is unsatisfactory.

It is the purpose of this report to develop a method for predicting the escape velocity that will be satisfactory for values of the ratio of the mass of the charge to that of the projectile as large as unity. The theory follows Hugoniot in treating the efflux of gas after departure of the projectile as equivalent to the emptying of a reservoir through a nozzle, values of quantities at the breech being used for values in the reservoir. In order, however, that the theory may hold for high values of ε, values at ejection of the ratios of mean pressure and mean density to breech values are derived from the Pidduck-Kent special solution for the motion of the powder gas.

The resulting equation is applied to the 240 mm howitzer, Model 1918, to the 3" seacoast gun, Model 1898, and to small arms. The agreement for the large caliber guns is within 3% on the average, the value of ε being as large as 1/3 for the 3" gun. It is shown, moreover, that the extension to values of ε as large as unity does not involve more than about a 5% correction; thus even if the latter should be in error by 20%, only 1% possible error would be added. The equation is thus expected to be equally satisfactory for $\varepsilon = 1$. A complete list of the main equation and of the auxiliary formulas is given in the summary.

As by-products there are developed formulas for the explicit calculation of the main parameter a_o of the Kent solution as a function of ε and a numerical verification that the value valid for small ε, viz. $\frac{1+\varepsilon/2}{1+\varepsilon/3}$, for the ratio of mean pressure to breech pressure before ejection is still accurate even for rather large values of ε. These results should be useful in ordinary pre-ejection interior ballistics.

TABLE OF CONTENTS

I. Introduction

1. The Maximum Velocity of Free Recoil

The design of a system for damping the recoil of a gun requires a knowledge of the maximum velocity that the gun would acquire if allowed to recoil with no retarding force whatever. The latter quantity, called the maximum velocity of free recoil, we shall denote by the symbol V_{rf}. (The subscript r denotes "recoil" and the subscript f "final value".) It is commonly determined by one of two methods. The first consists in mounting the gun on rollers, designed to be as friction-free as possible, and determining directly the actual velocity of recoil as a function of time. For this purpose some form of velocimeter or chronograph is used. The second method consists in mounting the gun as a ballistic pendulum and determining the rise in the center of gravity of the whole system when the gun is fired. From this measurement the maximum velocity is easily calculated for the free recoil of the compound system. To determine the maximum velocity of free recoil of the gun alone one then assumes that the recoil momentum is independent of the mass of the recoiling parts. Such an assumption is equivalent to the assumption that the gas pressure inside the gun, as a function of space and time, is independent of the mass of the recoiling parts. The latter assumption is entirely acceptable, in view of the known smallness of the effect of recoil on interior ballistic processes. If M_p denotes the total mass that recoils in the pendulum experiment and M_r denotes the mass of the recoiling parts of the gun in an ordinary firing, the desired value of V_{rf} is therefore given by the product of the pendulum velocity and the factor M_p/M_v. The roller method is used only for large caliber guns, while the pendulum method is practicable only for small calibers, although it has been used for guns as large as the three-inch.

2. The Effective Velocity of Escape of the Powder Gas.

The results of such experiments are commonly presented by giving the value of a quantity called "the effective velocity of escape of the powder gas", denoted by V_e. This quantity is defined by the following equation:

$$M_r V_{rf} = MV_m + CV_e, \tag{1}$$

where:

$M_r \equiv$ mass of recoiling parts

$M \equiv$ mass of projectile

$C \equiv$ mass of powder charge

$V_{rf} \equiv$ maximum velocity of free recoil

$V_m \equiv$ muzzle velocity of projectile

Since Eq. (1) obviously refers to a conservation of momentum, we may interpret V_e as follows. The final momentum $M_r V_{rf}$ of the freely recoiling gun must be equal to the sum of the momentum MV_m of the projectile, the final momentum of the escaping powder gas, and the momentum imparted to the surrounding air by the blast. Thus CV_e represents the sum of the last two momenta. We now make the reasonable assumption that $M_r V_{rf}$ and MV_m are independent of the state of the atmosphere, so that their values would be unchanged if the firing were into a vacuum. It follows that V_e represents the final value that would be achieved on the average by the axial component of powder gas velocity, if the firing were into a vacuum. The quantity V_e is not the same as the average gas velocity at the muzzle after ejection of the projectile, since the powder gas speeds up on leaving the gun, just as would the gas in a nozzle in passing from the convergent to the divergent portion. Furthermore we shall calculate V_e not by any direct consideration of processes at the muzzle, [illegible] of V_{rf} and use of Eq. (1). For these reasons, although we have interpreted Eq. (1) as referring to a conservation of momentum, it appears desirable to adhere to the position taken above, viz. that Eq. (1) is simply a definition of V_e.

The importance of the quantity V_e lies in the fact that it is presumably independent of the mass M_r of the recoiling parts. One can understand this statement in the light of the following remarks. In the usual case where $M_r >> M$, it is known that recoil has little effect on processes inside the gun. Thus gas pressure, as a function of space and time, will depend only very weakly on the ratio M/M_r, so that the momenta $M_r V_{rf}$ and MV_m will not depend appreciably on M_r. Then from Eq. (1) the escape velocity V_e will be independent of M_r. The lack of influence of M_r on V_e makes the latter a useful quantity for the expression of results on recoil, since various values of M_r may be contemplated in a design. Although V_e is thus independent of M_r, we shall see, however, that it may depend on other variables, so that the common practice in ordnance engineering of using a universal value* for V_e is therefore not justified.

3. The Quantity K

We define K by:

$$K \equiv V_e/V_m \tag{2}$$

Then, from (1) and (2):

$$M_r V_{rf} = (M + KC)V_m \tag{3}$$

II. Theory of the Velocity of Free Recoil

1. The Velocity of Free Recoil Before Ejection

Let V_{rm} be the velocity of the recoiling parts when the base of the projectile is flush with the muzzle. For values of $\varepsilon = C/M$ that are not too large (e.g. $\varepsilon < 1/3$), all theories of the motion of the powder gas give the value $1/2\ CV_m$ as the momentum of the powder gas. The recoil momentum at this stage is then given by:

$$M_r V_{rm} = (M + \tfrac{1}{2}\ C)V_m \tag{4}$$

2. Additional Velocity of Free Recoil After Ejection

A. Discussion of the Problem

After the projectile has left the gun, there are still several forces acting that tend to change the momentum of the freely recoiling gun. These are

(a) The force due to gas pressure on the breech.
(b) The force due to gas pressure on the muzzle ring, i.e. the force due to the pressure of escaping gas acting on an area $\frac{\pi}{4}(D_2^2-D_1^2)$, where D_2 is the outside diameter at the muzzle and D_1 the inside diameter.
(c) The drag due to gas friction on the walls of the chamber and the bore.
(d) The chambrage force, i.e. the force due to gas pressure acting on the cone joining the chamber or cartridge case with the bore. Let Fig. 1 schematize roughly the chamber (or cartridge case) and the bore of a gun.

* E.g. the well known texts by Tschappat, MacFarland, and Hayes, all suggest the value V_e = 4700 ft/sec.

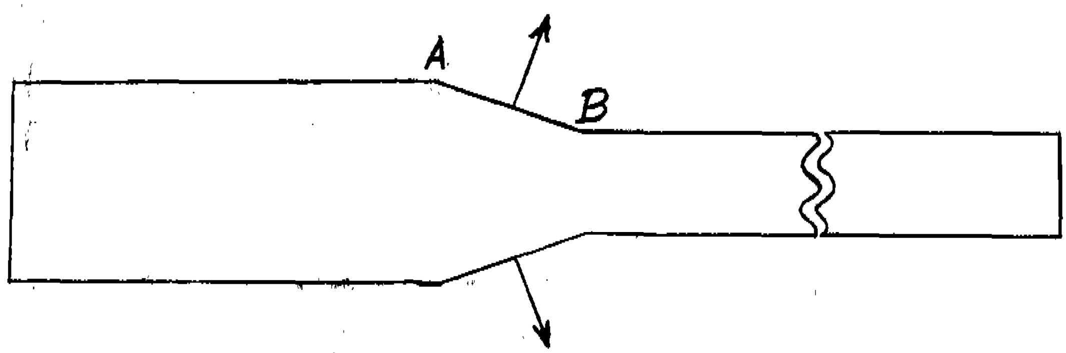

Fig. 1. Schematic representation of chamber and bore.

The chambrage force, which is the resultant force due to gas pressure acting on the cone AB, is axial and points toward the muzzle in the usual case where the chamber exceeds the bore in diameter.

Of these four forces (a) and (b) tend to increase the recoil momentum and (c) and (d) to reduce it. The pressure on the breech is the main factor, however, so that (b), (c), and (d) will be neglected. Effects (c) and (d) will presumably be of importance only for small arms. (In large caliber guns gas friction is usually considered unimportant, and the ratio of chamber diameter to bore diameter is close to unity). In any case (b) will help to cancel the effects (c) and (d).

In the following analysis we let A denote the cross-sectional area of the bore, neglecting any departure of chamber cross-section from this value*, p the breech pressure, and t the time measured from ejection. The recoil momentum added after ejection is then

$$M_r(V_{rf} - V_{rm}) = A\int_0^\infty p\ dt \tag{5}$$

The problem is thus reduced to the calculation of breech pressure p as a function of time, after ejection of the projectile.

B. The Hugoniot Theory

Following a suggestion by Corner**, we shall use a method due to Hugoniot*** for the calculation of p as a function of time. In this method one treats the efflux of gas after shot ejection as equivalent to the emptying of a reservoir through a

* In this connection cf. Section III 4, page 17.
** J. Corner, A.C. 4502, I.B. 201, Gn. 277
*** Hugoniot, Comp. Rend. 103, 1002 (1886)

nozzle. In the usual resevoir problem one takes the gas velocity to be zero in the resevoir; we shall therefore interpret resevoir values as referring to the breech. Let p and ρ denote the resevoir or breech values of pressure and density at time t and $\bar{p}$ and $\bar{\rho}$ their space-mean values. The subscript e will denote initial or shot ejection values. Let us now define r_1 and r_2 by:

$$\rho = r_1 \bar{\rho} \tag{6}$$

$$p = r_2 \bar{p} \tag{7}$$

We shall assume r_1 and r_2 to remain constant during efflux and therefore equal to their shot ejection values, which can be calculated from the theory of flow before ejection. With neglect of corrections due to gas imperfection, the rate of mass efflux is given by:

$$\varphi = A \; \gamma^{1/2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} (p\rho)^{1/2}, \tag{8}$$

where γ is the ratio of specific heats of the powder gas. Eq. (8) holds only when the ratio of external pressure to breech pressure is less than a certain critical value, but by the time the breech pressure has fallen so low that this critical value is exceeded, the gun is practically emptied of gas, so that no appreciable error can occur on this account. The pressure p and density ρ are connected by the adiabatic relation:

$$p\rho^{-\gamma} = p_e \rho_e^{-\gamma} \tag{9}$$

If X denotes the ratio of the total volume from breech to muzzle to the area A of the bore, the differential equation for emptying is:

$$A \, X \frac{d\bar{\rho}}{dt} = - \varphi \tag{10}$$

Combination of Eqs. (6), (8), (9), and (10) and integration with the initial condition $p = p_e$ gives:

$$p = p_e \left(1 + \frac{t}{\tau}\right)^{\frac{-2\gamma}{\gamma-1}}, \tag{11}$$

where $$\tau \equiv (X/r_1) \frac{2}{\gamma-1} \gamma^{-1/2} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} (\rho_e/p_e)^{1/2}, \quad (12$$

a time constant that governs the rate of fall of pressure after ejection. Let T_e denote the space-mean absolute temperature of the powder gas at shot ejection, and R_1 the gas constant per unit mass. Then, from the equation of state:

$$\overline{p}_e = \overline{\rho}_e R_1 T_e \tag{13}$$

Also, from the definition of $\overline{\rho}$,

$$\overline{\rho}_e = C/(AX) \tag{14}$$

Integration of (11), with use of (12), (13), and (14), gives for the additional recoil momentum after ejection:

$$A\int_0^\infty p\,dt = r_1^{-1/2}\; r_2^{1/2}\; f(\gamma)C(R_1T_e)^{1/2} \quad , \tag{15}$$

where

$$f(\gamma) \equiv \gamma^{-1/2}\left(\frac{\gamma+1}{2}\right)^{\frac{3-\gamma}{2(\gamma-1)}} \tag{16}$$

The usual uniform density theory for gas flow before ejection gives $r_1=1$ and $r_2 = (1+\varepsilon/2)/(1+\varepsilon/3)$. These values will be valid for values of $\varepsilon \equiv C/M$ that are not too large, say for $\varepsilon < \frac{1}{3}$. We shall consider later what modifications of these values may be necessary for larger values of ε.

C. Estimation of the Ejection Temperature T_e

During the travel of the projectile in the gun energy is lost from the system (powder gas plus projectile) through two effects, viz. by heat transfer from the hot powder gas to the cooler walls of the gun and by bore friction of the projectile. Let k denote the ratio of the total energy lost in these ways (at the moment when the projectile base has just reached the muzzle) to the kinetic energy $1/2\ MV_m^2$ of the projectile. The kinetic energy of the powder gas is given by $1/2\ C/\delta\ V_m^2$, where δ has the value 3 for sufficiently small values of ε; its value will be given later for larger values. The energy equation of interior ballistics then gives

$$R_1T_e = \lambda - \frac{\gamma-1}{2}\left(\frac{1+k+\varepsilon/\delta}{\varepsilon}\right) V_m^2 \quad , \tag{17}$$

where λ denotes the specific force of the powder. The best estimate available for k is given by a statistical analysis of certain firing records for single-perforated powder. Such an analysis showed that k depends significantly on w, D, Δ, and T_o only, where

$w \equiv$ web thickness of the powder

$D \equiv$ effective caliber of the gun $= \sqrt{\frac{4}{\pi} A}$

$\Delta \equiv$ density of loading

$T_o \equiv$ adiabatic flame temperature of the powder

A least square fit of log k as a linear function of the logarithms of w, D, Δ, and $\frac{T_o}{1000} - 1$ gave coefficients close to +1, -1, -1, and $-\frac{1}{2}$ respectively. The final formula obtained was:

$$k = \frac{25.8\ w/D}{\Delta\sqrt{\frac{T_o}{1000} - 1}} \tag{18}$$

In Eq. (18) w and D are to be expressed in the same units, Δ in gm/cm^3, and T_o in degrees Kelvin, while k is a pure number. Eqs. (17) and (18) then permit the calculation of $R_1 T_e$.

3. The Escape Velocity for Small Values of the Ratio of Charge to Projectile Mass

Combination of Eqs. (3), (4), (5), and (15), with use of the values $r_1 = 1$ and $r_2 = (1 + \varepsilon/2)/(1 + \varepsilon/3)$, gives:

$$K \equiv V_e/V_m = \frac{1}{2} + \left(\frac{1 + \varepsilon/2}{1 + \varepsilon/3}\right)^{1/2} f(\gamma)(R_1 T_e)^{1/2}/V_m\ , \tag{19}$$

where $f(\gamma)$ is given by (16) and $R_1 T_e$ by (17) and (18).

4. Extension to Values of ε as Large as Unity

We now consider what changes have to be made in (19) for values of ε that are not small compared to 1. For this purpose we use the Pidduck*-Kent** solution for the motion of the powder gas before ejection. Reference is made especially to the treatment as developed by Kent. For convenience let

$$\frac{1}{\gamma - 1} \equiv n \tag{20}$$

* Love and Pidduck, Phil. Trans. Roy. Soc. 222, 167(1922)
** R. H. Kent, Physics 7, 319 (1936).

Then the momentum of the powder gas just before ejection is given by:

$$\text{Momentum} = \frac{1}{2}\, C\, V_m h, \tag{21}$$

where h is a correction factor close to unity even for $\varepsilon = 1$. It has the value:

$$h = \frac{2}{\varepsilon}\left[(1-a_o)^{-n-1} - 1\right], \tag{22}$$

where a_o is the solution of the equation

$$2a_o(n+1)S(1-a_o)^{-n-1} = \varepsilon\ , \tag{23}$$

where

$$S \equiv \int_0^1 (1 - a_o\mu^2)^n\, d\mu\ , \tag{24}$$

a function of a_o and n.

Expansion gives:

$$h = 1 - \frac{n}{6}\, a_o + \frac{n}{90}(n-6)a_o^2 + \ldots \tag{25}$$

We are concerned with values of n about equal to 4 and with values of a_o less than 0.1 (for $\varepsilon = 1$, $a_o \approx 0.1$).

Thus, to 1 part in 1000:

$$h = 1 - \frac{n}{6}\, a_o \tag{25.1}$$

(The calculation of a_o will be considered shortly.)

Eq. (4) then becomes

$$M_r V_{rm} = (M + \frac{h}{2}\, C)\, V_m \tag{26}$$

It leads to replacement in (19) of the first term $\frac{1}{2}$ by h/2.

We now have to consider r_1 and r_2 in (15). From Kent's solution, we derive:

$$\frac{1}{r_2} = \frac{2(n+1)}{2n+3} S(1 + \frac{a_o}{\varepsilon}) \tag{27}$$

With the use of (23), (24), and (27) we construct the following table for the case n = 4 (for integral values of n, S is a polynomial in a_o):

Table I

a_o	S	ε	$\frac{1}{r_2}$	$\frac{1+\varepsilon/3}{1+\varepsilon/2}$ (Value of $1/r_2$ for small ε)	$1-\varepsilon/6$ (Value of $1/r_2$ for very small ε
0.08	0.900725	1.0933	0.8788	0.8822	0.8178
0.10	0.878106	1.4871	0.8520	0.8578	0.7522
0.20	0.776940	4.7421	0.7361	0.7655	0.2906

As ε diminishes, $1/r_2$ approaches the value $\frac{1+\varepsilon/3}{1+\varepsilon/2}$, which is labeled in Table I as the value for "small ε". Of course, if ε is sufficiently small $\frac{1+\varepsilon/3}{1+\varepsilon/2}$ can be expressed as $1 - \varepsilon/6$, which is accordingly labeled in Table I as the value of $1/r_2$ for "very small ε" Inspection of Table I shows that the value for "very small ε" approximates $1/r_2$ very poorly for values of ε greater than unity, but that the value for "small ε" is only 4% in error for a value of ε as large as 4.7 and only 0.4% in error for ε = 1.1. Thus for $\varepsilon \leq 1$ we have, correct to better than 0.2%:

$$r_2^{1/2} = \left(\frac{1+\varepsilon/2}{1+\varepsilon/3}\right)^{1/2} \tag{28}$$

For r_1 Kent's solution gives:

$$\frac{1}{r_1} = S = \int_0^1 (1-a_o\mu^2)^n \, d\mu \tag{29}$$

Expansion gives $$\frac{1}{r_1} = 1 - \frac{n}{3} a_o + \frac{n(n-1)}{10} a_o^2 + \ldots, \tag{30}$$

so that $$r_1^{-1/2} = 1 - \frac{n}{6} a_o + \frac{n}{360}(13n - 18)a_o^2 + \ldots \tag{31}$$

For n = 4 and ε = 1.1 (so that $a_o \approx 0.08$), the quadratic term amounts only to 0.2%. Thus, accurately enough for all values of $\varepsilon \leq 1$, we have;

$$r_1^{-1/2} = 1 - \frac{n}{6} a_o = h, \text{ by } (25.1) \tag{32}$$

The value of δ in (17) is given exactly by:

$$\frac{1}{\delta} = \frac{1}{2n+3}\left[\frac{1}{a_o} - \frac{2(n+1)}{\varepsilon}\right] \tag{33}$$

We now consider the calculation of a_o. From Eqs. (23) and (24) one can obtain ε as a series in a_o and can then invert the series (by Taylor's theorem) to obtain a_o as a series in ε. Unfortunately this series converges so slowly that, even if terms are kept through ε^5, the error amounts to 4% for $n = 4$ and $\varepsilon = 1.1$; such an error is too great for use of the series in (33), where accuracy is lost by subtraction. Reference is made to the Appendix for derivation of the following procedure. For $n = 4$ and ε as large as 1.3 it gives a_o correct to 1 part in 7000.

Let
$$\frac{\varepsilon}{2(n+1)} \equiv E \tag{34}$$

$$\frac{a_o}{1-a_o} \equiv b \tag{35}$$

Calculate
$$b_1 \equiv \frac{3}{4n}\left[\left(1 + \frac{8}{3} nE\right)^{1/2} - 1\right] \tag{36}$$

and
$$E' \equiv E - \frac{4}{15} n(n-1) b_1^3 \tag{37}$$

Then
$$b = \frac{3}{4n}\left[\left(1 + \frac{8}{3} nE'\right)^{1/2} - 1\right] \tag{38}$$

and
$$a_o = \frac{b}{1+b} \tag{39}$$

On replacing the 1/2 in (19) by $h/2$, as mentioned after Eq. (26), and comparing (15), (28), and (32) with (19), we have as our corrected formula:

$$K \equiv \frac{V_e}{V_m} = \frac{h}{2} + h\left(\frac{1+\varepsilon/2}{1+\varepsilon/3}\right)^{1/2} f(\gamma)(R_1T_e)^{1/2}/V_m \tag{40}$$

where h is given by (25.1), $f(\gamma)$ by (16), R_1T_e by (17),(18), and (33), and a_o by Eqs. (34) to (39). Any uncertainty as to the validity of Eq. (40) would arise from the second, i.e. the post-ejection term. We shall therefore compare it with experiment by multiplying the second term by a "fudge factor" F and calculating in each case the value of F that has to be assigned to obtain agreement with experiment. We have then:

$$K = \frac{h}{2} + F\, h \left(\frac{1 + \varepsilon/2}{1 + \varepsilon/3}\right)^{1/2} f(\gamma)(R_1 T_e)^{1/2}/V_m \quad . \tag{41}$$

III. Comparison with Experiment

1. Kent's Firings in the 240mm Howitzer, Model of 1918 M1.

These firings are reported in Ordnance Technical Notes No. 6*. The following data are relevant: chamber volume = 1790 in^3, A = 71.11 in^2, $D = (\frac{4}{\pi} A)^{1/2}$ = 9.515 in, pyro powder. Other data follow round by round:

Table II

Rd. No.	w(in)	ε	Δ(gm/cm^3)	V_m(f/s)	V_e(f/s)	K
18	0.036	0.02825	0.1546	822	4082	4.966
19	0.036	0.04085	0.2242	1021	3782	3.704
9	0.0488	0.06314	0.3450	1056	4300	4.072
11	0.0488	0.10453	0.5751	1546	4400	2.846

For γ, T_o, and λ we use nominal values for pyro powder, given by Hirschfelder, Kershner, and Sherman.** These are γ = 1.228, T_o = (2610 + 5/w) degrees Kelvin, and λ = (303,000+600/w)ft.lbwt/lb, where w denotes the web thickness in inches. (The correction terms in 1/w allow for the effect of moisture and volatiles). We then have n = 1/0.228 and $f(\gamma)$ = 1.3728. Table III then follows, where B denotes the coefficient of F in Eq. (41).

Table III

Rd. No.	k	a_o	h/2	$1/\delta$	$(R_1 T_e)^{1/2}$	B	F
18	0.4776	0.00259583	0.499	0.3323	2500 f/s	4.176	1.07
19	0.3293	0.00373645	0.499	0.3319	2528	3.402	0.94
9	0.2932	0.00573059	0.498	0.3312	2735	3.559	1.00±0.0
11	0.1759	0.00935453	0.497	0.3270	2642	2.349	1.00∓0.0
						Mean =	1.00

*Ordnance Technical Notes No. 6, "Experiments in Interior Ballistics", Office of the Chief of Ordnance, U.S. Army, 1925.

**NDRC Armor and Ordnance Report No. A-204, page 87.

The values given for a_o are of course not really accurate to so many significant figures, but they are accurately consistent with the values of ε and γ. Thus loss of accuracy by subtraction is avoided in Eq. (33) for $1/\delta$. For rounds 9 and 11 the values of V_e were given only to the nearest hundred feet per second, i.e. to 1/2 part in 44 or to 1%, so that the corresponding values of F are uncertain by 1%. The mean value of F comes out exactly 1.00.

2. Pastoriza's Firings in the 3" Seacoast Gun, Model of 1898

These ballistic pendulum firings are reported in the First Progress Report on Ordnance Board Program No. 2337-2, "Gas Deflector for Reducing Recoil in Guns", October 25, 1919. The data follow:

Projectile mass M = 15.00 lb.

From No. 1676, "Table of U.S. Army Cannon, Carriages, and Projectiles", March 24, 1904, Revised Jan. 15, 1924:

Chamber volume = 200 in^3

$A = 7.279\ in^2$

$\therefore D = 3.044\ in$

No information was given about the powder. All powders at that time, however, were pyro powders and the above cannon table gives as the typical web thickness w = 0.047". Hirschfelder's nominal values are then $\gamma = 1.228$, $T_o = 2716$ degrees Kelvin, and $\lambda = 316{,}000$ ft.lb.wt/lb. = $(1017) \cdot 10^4\ ft^2\ sec^{-2}$. Various charges were fired without a gas deflector, but muzzle velocities were taken only on rounds 16, 17, and 18 with a charge C = 5.00 lb. The mean value was $V_m = 1/3(2657 + 2645 + 2660) = 2654$ ft/sec. Rounds 14, 15, 19, and 27 were the only successful* rounds at a charge of 5 lb. without a gas deflector, but no muzzle velocities were taken on these rounds. We shall, however, use the above value $V_m = 2654$ ft/sec for these rounds also. Various recoiling masses were used. As discussed in the introduction, however, the recoil momentum is expected to be independent of M_r, so that it is appropriate to use an average value. The values of $M_r V_{rf}$ in slug ft/sec were respectively 1970, 1942, 1970, and 1974, the average being 1964 slug ft/sec or 63,190 lb. ft/sec. Eq (1) then gives:

$$5\ V_e + 15(2654) = 63{,}190, \tag{42}$$

leading to $V_e = 4676$ ft/sec and $K = \frac{4676}{2654} = 1.762$.

Calculation then gives $\varepsilon = 1/3$, $\Delta = 0.6921\ gm/cm^3$,

* i.e. successful in giving reliable values for recoil velocity.

$k = 0.4393$, $a_o = 0.0276919$, $\frac{h}{2} = 0.490$, $1/\delta = 0.3224$, $(R_1T_e)^{1/2} = 2538$ ft/sec, $B = 1.318$, and $F = 0.97$. The check is again very good.

3. Small Arms

For small arms we use some ballistic pendulum firings reported by Kent and Hitchcock.* These firings were all with IMR powder for which the nominal constants are $\gamma = 1.246$, T_o=2795 degrees Kelvin, $\lambda = 334{,}000$ ft.lb.wt/lb., $f(\gamma) = 1.3547$.

A. Caliber 0.50 Machine Gun (M1921 A.A.)

Data:	Calculated:
$w = 0.022$ in	$\varepsilon = 0.3403$
$V_m = 2500$ ft/sec	$\Delta = 0.922$ gm/cm³
$V_e = 4679$ ft/sec	$k = 0.908$
$C = 245$ grains	$a_o = 0.0299791$
$M = 720$ grains	$\frac{h}{2} = 0.490$
$A = 0.2011$ in²	$1/\delta = 0.3224$
$D = 0.506$ in.	$(R_1T_e)^{1/2} = 2488$ ft/sec
Chamber volume = 1.05 in³	$B = 1.354$
	$K = 1.872$
	$F = 1.02$

The agreement is better than would be expected.

B. Caliber .30

Data:	Calculated:
$w = 0.012$ in	$\varepsilon = 0.2895$
$V_m = 2600$ ft/sec	$\Delta = 0.797$ gm/cm³
$C = 49.8$ grains	$k = 0.947$
$M = 172$ grains	$a_o = 0.0259090$
$A = 0.07355$ in²	$\frac{h}{2} = 0.491$
$D = 0.306$ in	$1/\delta = 0.3239$
Chamber volume = 0.247 in³	$(R_1T_e)^{1/2} = 2211$ ft/sec
	$B = 1.157$

* R. H. Kent and H. P. Hitchcock, Ballistic Research Laboratory Report No. 171 (original date April 5, 1929, revised 18 January 1940).

We obtain

$$1.157\ F = \frac{V_e}{2600} - 0.491 \qquad (V_e \text{ in ft/sec}) \tag{43}$$

The following table gives values of V_e from the above report with the resulting values of F.

Table IV

Gun	Special Remarks	V_e ft/sec	F
Automatic Rifle			
Cal. 0.30 M 1915	Middle Gas Port	4115	0.94
	Small Gas Port	3710	0.81
Rifle Cal.0.30 M1903	First Barrel	3902	0.87
	Second Barrel	3720	0.81
Machine Gun Cal.0.30 M1917		3839	0.85
		Mean =	0.86

4. Discussion of the Results

Table V summarizes the mean values of F.

Table V

Gun	Mean F
240mm Howitzer Model of 1918 M1	1.00
3" Seacoast Gun Model of 1898	0.97
Caliber 0.50	1.02
Caliber 0.30	0.86

Neglect of gas friction and of the chambrage effect is expected to be the most serious in the case of the smallest calibers* with a corresponding overestimate of the recoil momentum after ejection and a resulting smaller value for F. It is therefore very reasonable that the value for the caliber 0.30 comes out somewhat lower than the others. The larger value 1.02 for the caliber 0.50, however, is unexpected, but it is based on only three rounds as compared with thirteen for the caliber 0.30.

It is appropriate at this point to say a little more about neglect of the chambrage effect. In small arms the ratio of the cross-section of the chamber to that of the bore may amount to as much as 2. It might therefore seem surprising that neglect of the retarding force on the joining cone does not lead to very serious error in such cases. The answer is that our assumption of a uniform cross-section equal to that of the bore largely compensates for this effect, since it assigns too small a value to the area of the breech. If there were no pressure drop between the breech and the entrance to the bore, the compensation would be exact. Actually the latter drop is only a small fraction of the total pressure drop from breech to muzzle, so that the compensation, although not complete, should be almost so.

For guns other than small arms it appears that the value unity should be satisfactory for F. Thus we may use Eq. (40) with no empirical correction factor.

IV. Summary

For guns other than small arms the effective velocity of escape V_e of the powder gas after ejection is given by the following equation:

$$K \equiv V_e/V_m = \frac{h}{2} + hf(\gamma) \left(\frac{1+\varepsilon/2}{1+\varepsilon/3}\right)^{1/2} (R_1T_e)^{1/2}/V_m \qquad (40)$$

$V_m \equiv$ muzzle velocty

$\varepsilon \equiv$ ratio of charge weight to projectile weight

$\gamma \equiv$ ratio of specific heats of the powder gas

$$h = 1 - \frac{n}{6}\, a_o, \text{ where } n \equiv 1/(\gamma-1) \qquad (25.1)$$

a_o is given by calculating the following quantities:

$$E \equiv \frac{\varepsilon}{2(n+1)} \qquad (34)$$

$$b_1 \equiv \frac{3}{4n} \left[(1 + \frac{8}{3}\, n\, E)^{1/2} - 1\right] \qquad (36)$$

$$E' \equiv E - \frac{4}{15}\, n(n-1)\, b_1^3 \qquad (37)$$

*The gas friction effect is expected to be more important with small calibers mainly because the small caliber guns are longer in calibers than the larger guns.

$$b = \frac{3}{4n} \left[\left(1 + \frac{8}{3} n E'\right)^{1/2} - 1\right] \tag{38}$$

Then

$$a_o = \frac{b}{1+b} \tag{39}$$

$$f(\gamma) \equiv \gamma^{-1/2} \left(\frac{\gamma+1}{2}\right)^{\frac{3-\gamma}{2(\gamma-1)}} \tag{16}$$

$$R_1 T_e \equiv \lambda - \frac{\gamma-1}{2}\left(\frac{1+k+\varepsilon/\delta}{\varepsilon}\right) V_m^2, \quad \text{where} \tag{17}$$

$\lambda \equiv$ specific force of the powder

$$k = \frac{25.8 \ w/D}{\Delta\left(\frac{T_o}{1000} - 1\right)^{1/2}} \tag{18}$$

$w \equiv$ web thickness of the powder

$D \equiv$ effective caliber $= \left(\frac{4}{\pi} \text{ bore cross-section}\right)^{1/2}$

(w and D are to be expressed in the same units)

$\Delta \equiv$ density of loading in gm/cm^3

$T_o \equiv$ adiabatic flame temperature of the powder in degrees Kelvin

$$1/\delta = \frac{1}{2n+3}\left(\frac{1}{a_o} - \frac{1}{E}\right)$$

Equation (40) has been shown to be satisfactory for large caliber guns for values of ε as large as 1/3. For values of ε less than 0.1 it is sufficiently accurate to use $h = 1$ and $\delta = 3$. The somewhat lengthy calculation of a_o can then be omitted. It was desired to obtain an equation which would be satisfactory for values of ε as large as 1. For such a value of ε, h is about 0.95 and the factor $\left(\frac{1+\varepsilon/2}{1+\varepsilon/3}\right)^{1/2}$ does not have to be modified; the extension to such values of ε does not therefore involve more than about a 5% correction. Even if the latter correction itself should be in error by as much as 20%, the added error would be only 1%. The equation is therefore expected to be satisfactory for values of ε as large as unity.

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APPENDIX: CALCULATION OF a_o FROM ε

From (23) and (24) we have

$$\varepsilon = 2(n+1)\ a_o(1-a_o)^{-n-1}\int_o^1 (1-a_o\mu^2)^n d\,\mu \tag{A1}$$

With the definitions $\dfrac{\varepsilon}{2(n+1)} \equiv E$ (A2)

and

$$\frac{a_o}{1-a_o} \equiv b, \tag{A3}$$

we have

$$E = b\int_o^1 \left(\frac{1-a_o\mu^2}{1-a_o}\right)^n d\mu \tag{A4}$$

Now

$$\frac{1-a_o\mu^2}{1-a_o} \equiv 1 + \frac{a_o}{1-a_o}(1-\mu^2) = 1 + b(1-\mu^2) \tag{A5}$$

Thus

$$E = b\int_o^1 \left[1 + b(1-\mu^2)\right]^n d\mu \tag{A6}$$

On expansion of the integrand in powers of b and integration, we have:

$$E=b\left[1 + \tfrac{2}{3}nb + \tfrac{4}{15}n(n-1)b^2 + \tfrac{8}{105}n(n-1)(n-2)b^3 + \tfrac{16}{945}n(n-1)(n-2)(n-3)b^4 + \ldots\right] \tag{A7}$$

For n=4 the series terminates with the last term written, and for $b = 0.1$, $E = 0.1\left[1 + 0.2666 + 0.032 + 0.0018{,}2857 + 0.0000{,}4063\right] = 0.13005359$, so that $\varepsilon = 10E = 1.3005359$.

Inspection shows that the error involved in stopping with the term $\frac{4}{15}\ n(n-1)b^3$ would be only 0.2%. This fact suggests using only the first three terms and solving the cubic for b.

For values of $b \leq 0.1$, i.e. for $\varepsilon \leq 1.3$, we may solve the cubic

$$b + \frac{2}{3}\ nb^2 + \frac{4}{15}\ n(n-1)b^3 = E \tag{A8}$$

to high accuracy by successive approximations.

In (A8) neglect of the term in b^3 gives as a first approximation for b:

$$b_1 = \frac{3}{4n}\left[\left(1 + \frac{8}{3} n E\right)^{1/2} - 1\right] \tag{A9}$$

Insertion of b_1 into the cubic term in (A8) gives

$$b + \frac{2}{3} n b^2 = E - \frac{4}{15} n(n-1)b_1^3 \equiv E' \tag{A10}$$

Solution of (A10) then gives:

$$b = \frac{3}{4n}\left[\left(1+ \frac{8}{3} n E'\right)^{1/2} - 1\right] \tag{A11}$$

Eq. (A11) completes the derivation of the method. For n = 4 and ε = 1.3005359 the value of b is known to be 0.10000000. The above approximate procedure gives b = 0.09998477 and a_o=0.09089650, to be compared with the accurate a_o = 0.1/1.1 = 0.0909,0909. The error is only 1 part in 7000. By Eq. (33) the accurate and approximate values of a_o give respectively for $1/\delta$ the values 0.3010 and 0.3011. The above procedure is thus adequate for the calculation of δ for values of $\varepsilon \leq 1$.

It may be asked why the procedure gives b correct to 0.015% when the cubic for E gives an error as large as 0.2%. The answer is clear. For a given value of E, b_1 is an overestimate for b because of neglect of powers higher than b^2. Use of b_1 in the cubic term thus overestimates the latter, thereby compensating for neglect of powers higher than the cubic.

TABLE OF SYMBOLS

SYMBOL	DEFINITION	PAGE
A	Cross-sectional area of the bore	6
a_o	Dimensionless parameter in Kent's special solution for the motion of the powder gas	10
B	Abbreviation for $h\left(\frac{1+\varepsilon/2}{1+\varepsilon/3}\right)^{1/2} f(\gamma)(R_1T_e)^{1/2}/V_m$	13
b	Abbreviation for $a_o/(1-a_o)$	12
b_1	First approximation for b	12
C	Mass of the powder charge	3
D	Effective caliber $\equiv (\frac{4}{\pi} A)^{1/2}$	9
E	Abbreviation for $\frac{\varepsilon}{2(n+1)}$	12
E'	Abbreviation for $E - \frac{4}{15} n(n-1)b_1^3$	12
F	Empirical factor to be adjusted to give correct recoil momentum after ejection	13
$f(\gamma)$	Abbreviation for $\gamma^{-1/2}\left(\frac{\gamma+1}{2}\right)^{\frac{3-\gamma}{2(\gamma-1)}}$	8
H_p	Abbreviation for $1 - \frac{n}{6}a_o$	10
K	Ratio of effective velocity of escape to muzzle velocity of projectile	5
k	Ratio of total heat loss before ejection to kinetic energy of projectile	8
M	Mass of projectile	4
M_r	Mass of recoiling parts	3
M_p	Total recoiling mass of ballistic pendulum	3
n	Abbreviation for $1/(\gamma-1)$	9
p	Pressure at the breech	6
$\bar{p}$	Space-mean value of pressure in the gun	7

SYMBOL	DEFINITION	PAGE
p_e	Breech pressure at ejection	7
$\overline{p}_e$	Space-mean pressure at ejection	18
R_1	Gas constant per unit mass	18
r_1	Ratio of density at the breech to space-mean density	7
r_2	Ratio of pressure at the breech to space-mean pressure	7
S	The integral $\int_0^1 (1-a_o\mu^2)^n \, d\mu$	10
T_o	Adiabatic flame temperature of the powder gas (°K)	19
T_e	Space-mean temperature of the gas at ejection (°K)	18
t	Time measured from ejection	6
V_e	Effective velocity of escape of the powder gas	3
V_m	Muzzle velocity of the projectile	4
V_{rf}	Maximum or final velocity of free recoil	3
V_{rm}	Velocity of free recoil when the projectile base is at the muzzle	9
w	Web thickness of the powder	19
X	Ratio of the total volume from breech to muzzle to the cross-sectional area of the bore	7
γ	Ratio of specific heats of the powder gas	7
Δ	Density of loading	19
δ	$1/\delta$ is the fraction of the mass of the powder to be added to the mass of the projectile for computation of the total kinetic energy of projectile plus powder	18
ε	Ratio of the mass of the powder to that of the projectile	5
λ	Specific force of the powder	18
μ	Dummy variable of integration in the integral S	10
ρ	Density of the powder gas at the breech	7
$\overline{\rho}$	Space-mean value of gas density in the gun	7
ρ_e	Breech density at ejection	7
$\overline{\rho}_e$	Space-mean density at ejection = $C/(AX)$	18
τ	Time constant for the rate of fall of pressure after ejection	17
φ	Rate of mass efflux of powder gas after ejection	7